Appendix E: Mathematical Symbols

Bold face $\begin{cases}\text{Vectors } \mathbf{V} \\ \text{Matrices } \mathbf{A}\end{cases}$

Superscripts $\begin{cases}\text{Powers as } a^n,\ \sin^n x = (\sin x)^n \\ \text{Inverse functions as } \sin^{-1} x = \text{arc}\sin x \\ \text{Order of differentiation as } x' = dx/dt;\ \ x'' = d^2x/dt^2\end{cases}$

Subscripts Position in a sequence, set, or matrix as $\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$

Parentheses $\begin{cases}\text{Aggregation as } (a+b) \\ \text{Argument of a function } f(t)\end{cases}$

Vertical bars $\begin{cases}\text{Absolute value, modulus, magnitude of vector} \\ \text{Determinant } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \text{ and sometimes matrices } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \\ \text{Evaluated at } f(x)\big|_a = f(a)\end{cases}$

Double bars Matrix $\|a_{ij}\| = \begin{Vmatrix} a_{11} & a_{12} \\ a_{11} & a_{21} \end{Vmatrix}$

Brackets	Ordered set $[x, y, z]$ or $[g_1, g_2, g_3, \cdots]$
Superscript line	Mean; complex conjugate
$=$	Equal to
$\neq$	Not equal to
$\approx$	Approximately equal to
$>$	Greater than
$<$	Smaller than
$\geqslant$	Greater than or equal to
$\leqslant$	Smaller than or equal to
$+$	Plus
$-$	Minus
$\pm, \mp$	Plus or minus, minus or plus
$\cdot$	Dot product
$\cup$	Union
$\cap$	Intersection
$\supset$	Contains; implies
$\rightarrow$	Approaches
$\leftrightarrow$	Transforms to (either way). Symbols are often indicated in lower case in the time (or space) domain and their transform in capital letters in the frequency (or wavenumber) domain.
/	Division, as $1/2$
*	Convolved with As superscript: complex conjugate, also indicated by superscript bar: $\overline{z}$
...	And so forth as $a_0, a_1, \ldots, a_n$
!	Factorial, as $4! = 4\times 3\times 2\times 1 = 24$
$\int_b^a$	Integral from a to b
$\oint$	Line integral around a closed loop
∞	Infinity
Δ	Difference; sizeable increment
∇	Del (a vector) $= \mathbf{i}\,\partial/\partial x + \mathbf{j}\,\partial/\partial y + \mathbf{k}\,\partial/\partial z$:

∇^2	Laplacian
δ	Very small increment
∂	Partial derivative, $\partial f(x,y)/\partial x$
Σ	Sum, as $\Sigma_{i=1}^{3} a_i = a_1 + a_2 + a_3$; $\Sigma =$ sum of all appropriate elements
Π	Product, as $\Pi_{i=1}^{3} a_i = a_1 a_2 a_3$
$\phi_{xy}(\tau)$	Correlation of x with y as function of time shift τ
σ	Standard deviation
ω	Angular velocity (frequency)
abs	Absolute
arc	Inverse, as arc $\sin x = \sin^{-1} x =$ angle whose sine is x
arg	Argument of
av	Average; also indicated by a superscript bar: $\overline{V}$
cis θ	$\cos\theta + i \sin\theta$
d	Differential
det	Determinant
div	Divergence
erf	Error function, as $\mathrm{erf}(x) = 2/\sqrt{\pi}\int_0^x e^{-v^2} dv$
erfc	Complementary error function $= 1 -$ erf
exp	Exponential function, as $\exp[x] = e^x$
Im	Imaginary part of
lim	Limit; $\lim_{a\to\infty} =$ limit as a approaches ∞
ln	Natural logarithm $= \log_e$ (log to the base e)
max	Maximum
$\mathrm{P}[E]$	Probability of E
$\mathrm{P}[E \cap F]$	Probability of both E and F
$\mathrm{P}[E/F]$	Probability of E given F
Re	Real part of
rms	Root-mean-square
sgn	Sign of as $\mathrm{sgn}(x) = +1$ if $x > 0$, -1 if $x < 0$

Cylindrical functions of order ν and argument x

- General $Z_\nu(x)$
- Bessel function of first kind $J_\nu(x)$
- Bessel function of second kind $N_\nu(x)$
- Modified Bessel function of second kind $K_\nu(x)$
- Hankel function of first kind $H_\nu^{(1)}(x)$
- Hankel function of second kind $H_\nu^{(2)}(x)$

Spherical function of order ν and argument x

$$z_\nu(x) = \left(\frac{\pi}{2x}\right)^{1/2} Z_{\nu+1/2}(x)$$

$$j_\nu(x); n_\nu(x); i_{gn}(x); k_{gn}(x); h_\nu^{(1)}(x); k_\nu^{(2)}(x)$$

Gamma function $\Gamma(x)$

Struve function $H_\nu(x)$

Modified Struve function $L_\nu(x)$

Error function $\mathrm{erf}(x)$

Complementary error function $\mathrm{erf}c(x)$

Legendre functions $P_\nu(x)$

Associated Legendre functions $P_\nu^m(x)$

Fourier transform pair $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{i\omega t}d\omega \leftrightarrow F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-i\omega t}dt$

Laplace transform $F(s)=L\{f(t)\}=\int_{0}^{\infty}f(t)e^{-st}dt$

Hilbert transform pair $\mathrm{Im}\{f(\omega)\}=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Re}(\alpha)}{\omega-\alpha}d\alpha \leftrightarrow \mathrm{Re}\{f(\omega)\}=\frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}(\alpha)}{\omega-\alpha}d\alpha$

Hankel transform pair $f_m(\rho)=\int_{0}^{\infty}F_m(\lambda)J_m(\lambda\rho)\lambda d\lambda \leftrightarrow F_m(\lambda)=\int_{0}^{\infty}f_m(\rho)J_m(\lambda\rho)\rho d\rho$

For definitions of functions and related matters, refer to Abramowitz, M. and Stegun, I. A., 1972, Handbook of mathematical functions: Dover Publications, Inc.